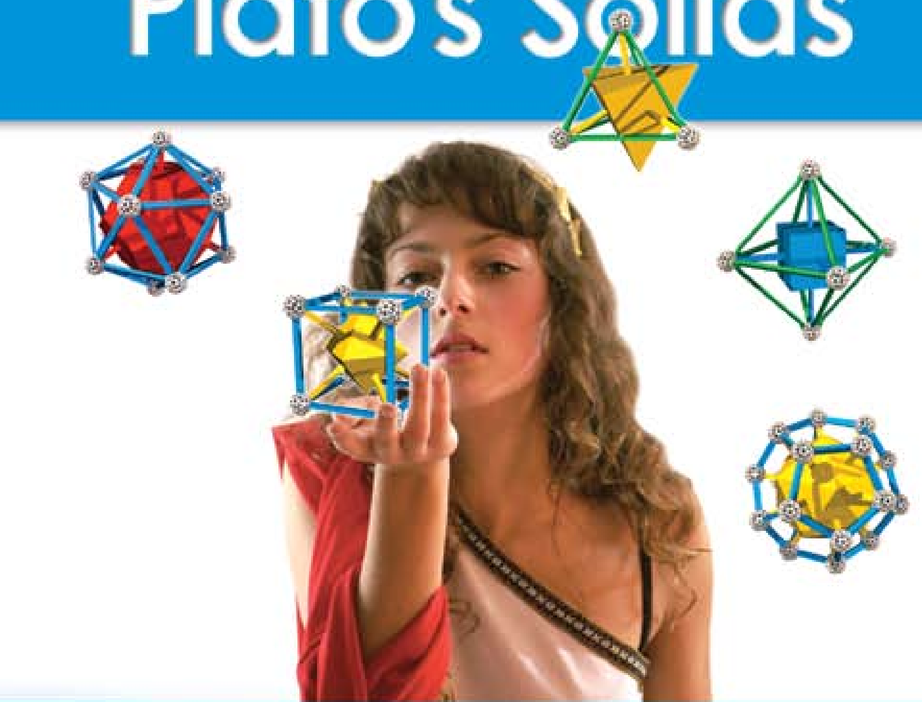
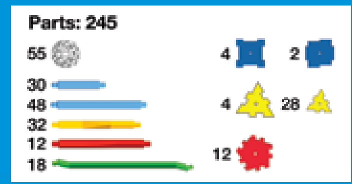
Plato's Solids





Includes 195 precision Zometool components, 50 foam dual pieces, and detailed instructions by Dr. Robert Fathauer

Why are there only 5 perfect 3D shapes? This secret was closely guarded by ancient Greeks, and is the mathematical basis of nearly all natural and human-built structures.

Build all five of Plato's solids in relation to their duals, and see how they represent the 5 elements:

- *the Tetrahedron (4-faces) = fire*
- *the Cube (6-faces) = earth*
- *the Octahedron (8-faces) = water*
- *the Icosahedron (20-faces) = air*
- *the Dodecahedron (12-faces) is the shape reserved for the cosmos!*

START HERE! Plato's Solids Instructions

You can build the five Platonic Solids, or *polyhedra*, and their *duals*.

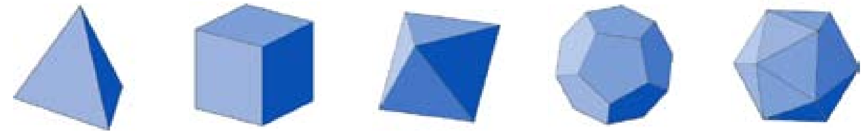
A polyhedron is a solid whose faces are polygons. Only five *convex regular* polyhedra exist (i.e., each face is the same type of regular *polygon*—a triangle, square or pentagon—and there are the same number of faces around every corner.)

If you put a point in the center of each face of a polyhedron, and connect those points to their nearest neighbors, you get its dual.

These center points are now the corners (or *vertices*) of the dual.

The Platonic Solids are named according to the number of faces (F) they possess. For example, "octahedron" means "8-faces." The number of Faces (F), Edges (E) and Vertices (V) for each solid are shown below. An edge is a line where two faces meet, and a vertex is a point where three or more faces meet.

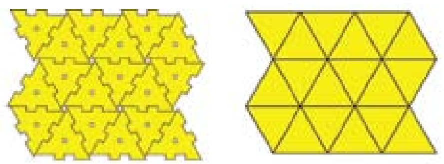
Each Platonic Solid has another Platonic Solid as its dual. The dual of the tetrahedron ("4-faces") is again a tetrahedron; the dual of the cube is the octahedron ("8-faces"), and vice versa. The dual of the dodecahedron ("12-faces") is the icosahedron ("20-faces"), and vice versa.



What do you notice about the number of faces and vertices for the solids and their duals? the edges?

Plane tessellations and polyhedra:

Before building the solids, look at how the foam pieces fit together on a flat surface. First try the small triangles. Notice the shape of a piece is different when it's flipped over. Be sure the same side is facing up (in a flat arrangement) or outward (in a polyhedron) for all of them. This is also true for the pentagons.

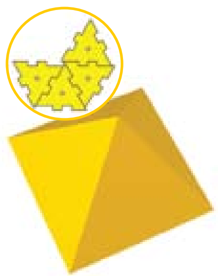


Ignoring holes and small gaps, they cover a flat surface. Equilateral triangles form one of the three regular tessellations (tilings).

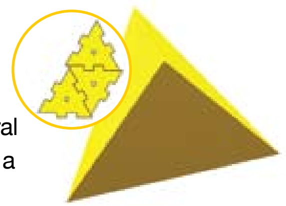
This means that each piece, or tile, is a regular polygon of the same type, and when put together they cover a flat surface (theoretically the infinite mathematical plane) without "real" gaps or overlaps. A regular tessellation is like a two-dimensional Platonic Solid.

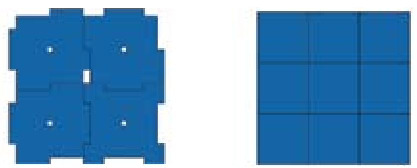
Now look at a point (vertex) where six triangles come together. Remove one of the six to leave a gap. If you bring the two edges of the gap together, you get a three-dimensional pyramid with a regular pentagon as its base. This pyramid is part of an icosahedron.

Next, lay them flat again and remove another triangle, so only four are left. If you bring these together to close the gap, you get a pyramid with a square base. This is half of an octahedron.



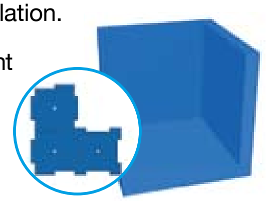
Finally, lay them flat again and remove another triangle, so only three are left. If you bring these together to close the gap, you get a pyramid with an equilateral triangle base. This is a tetrahedron.



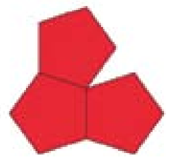


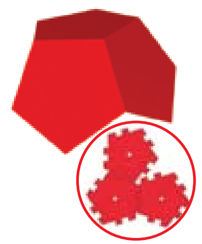
Next look at the square pieces. Notice they aren't all the same. Arrange four of them as shown. They don't quite lay flat, because the notches don't line up right in a flat arrangement. Regular squares do fit together without gaps or overlaps, and form the second regular tessellation.

Remove the upper right square as shown, and bring the two edges together to close the gap left by the removed square. This forms half a cube.



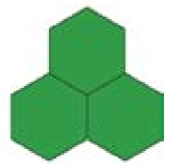
Finally, look at the pentagon pieces. If you fit three of these together on a flat surface, there is a small gap. Regular pentagons will not tessellate; i.e., they will not cover the plane without gaps or overlaps.



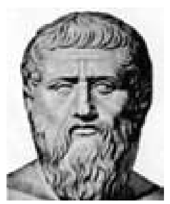


If you bring the two edges in the gap together, you will form part of a dodecahedron. You've now formed parts of each of the five Platonic Solids.

The final regular tessellation is made up of regular hexagons. Three hexagons fit together on a flat surface without a gap or overlap. Since at least three polygons must meet at the vertex of a polyhedron, there are no regular solids with hexagonal faces. Any regular polygon with more than six sides will have larger angles than a hexagon, so there are no regular solids whose faces have more than five sides.

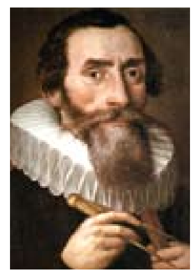


Solid history

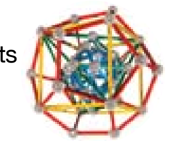


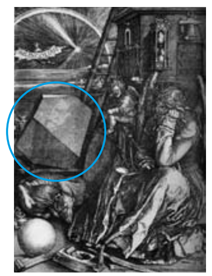
Plato was a Greek philosopher, mathematician, and teacher who lived around 429/423 – 348/347 BCE. His philosophical writings were a major influence in Western thought.

While Plato didn't discover the Platonic solids, he and his followers associated them with the four classical elements – earth with the cube, air with the octahedron, water with the icosahedron, and fire with the tetrahedron. The dodecahedron was reserved for the heavens.



The Platonic Solids and other polyhedra were widely studied during the Renaissance. Johannes Kepler was a German mathematician and astronomer who lived from 1571 – 1630. He discovered the laws of planetary motion that for the first time explained the orbits of the planets in our solar system. Earlier, he incorrectly associated the orbits of the six known planets with a nesting of the five Platonic Solids within a sphere.





Renaissance artists also studied the Platonics and other polyhedra: Albrecht Dürer (1471 – 1528) prominently featured a polyhedron in one of his most famous engravings, Melancolia I.

Tetrahedron

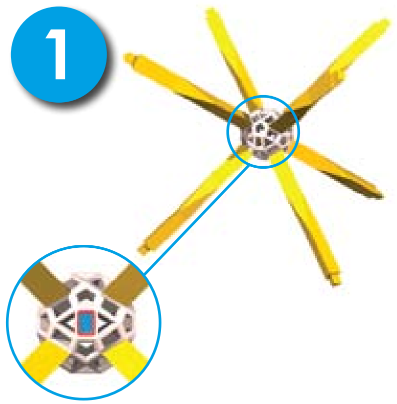
1


2


3

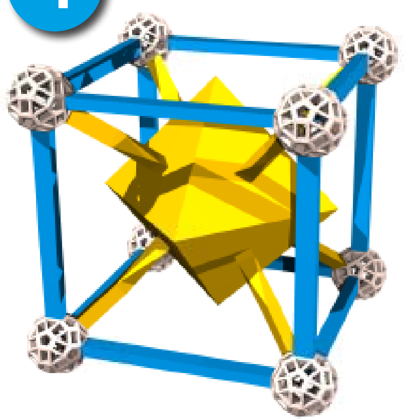

4


Cube/Octahedron

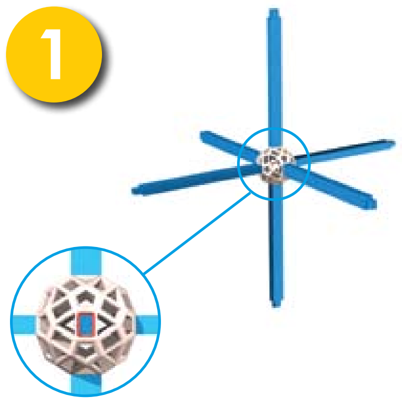
1


2


3


4


Octahedron/Cube

1


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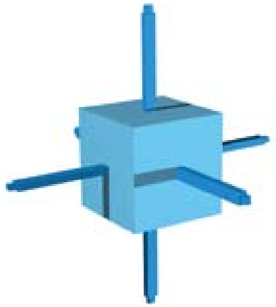

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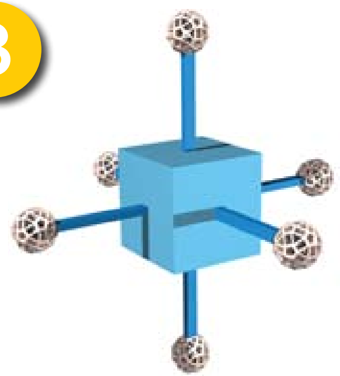

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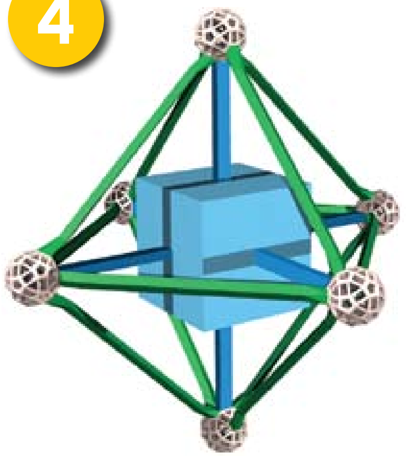
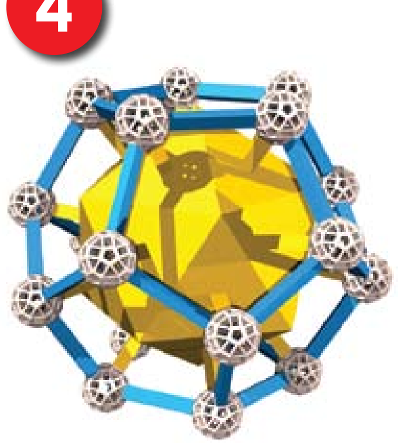


Dodeca/Icosahedron

1

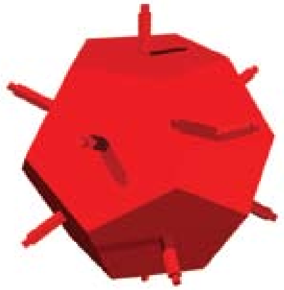

2


3


4


Icosa/Dodecahedron

1


2


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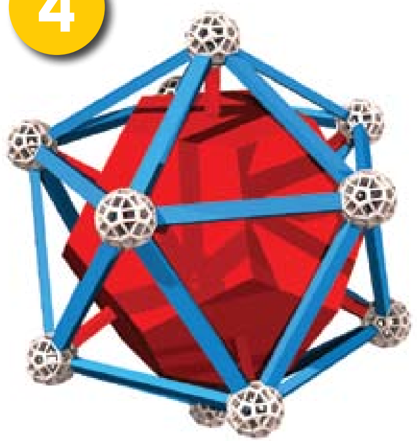
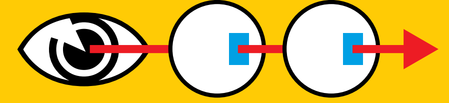
4


ZOMETOOL RULES!

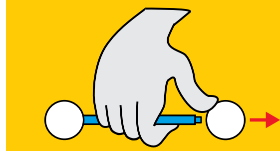
1 *If it works, it works perfectly.*

…and if it doesn't work, it doesn't work at all. Don't force Zometool components. You can bend a strut to fit it into a tight spot, but struts in finished models are always straight, never under tension.



Hint: you can tell which strut fits between two balls in a model by lining up the balls and looking through the holes. The holes show you the shape of the strut that fits!

2 *Don't break it apart; take it apart!*



Take Zometool models apart by grasping a strut with your fingers and pushing the ball straight off with your thumb. Twisting balls, pulling models apart or crushing them can cause parts to break!*

3 *Leave the place cleaner than you found it.*

It's always a good idea to clean up when you're done. If we work together, we can make the world better.

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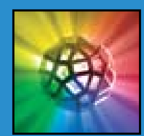

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