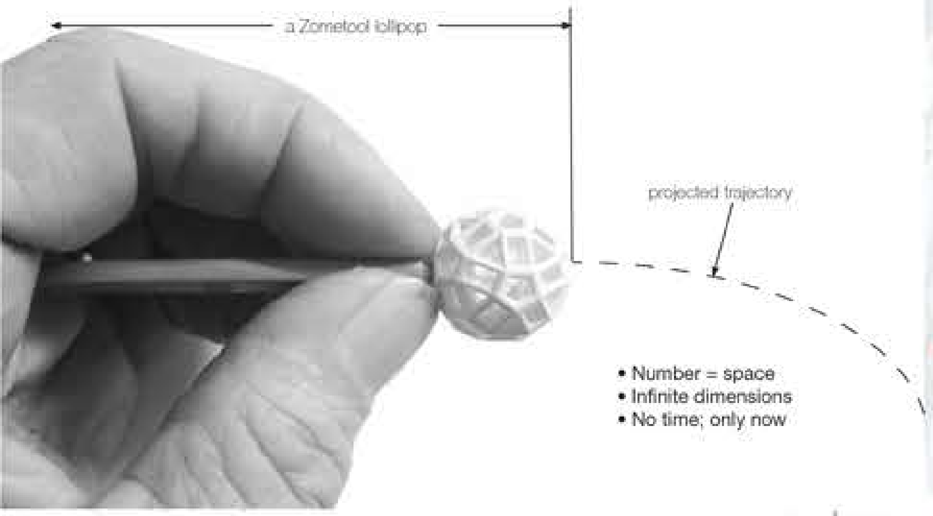
art and science at play

ZOMETOOL® rules

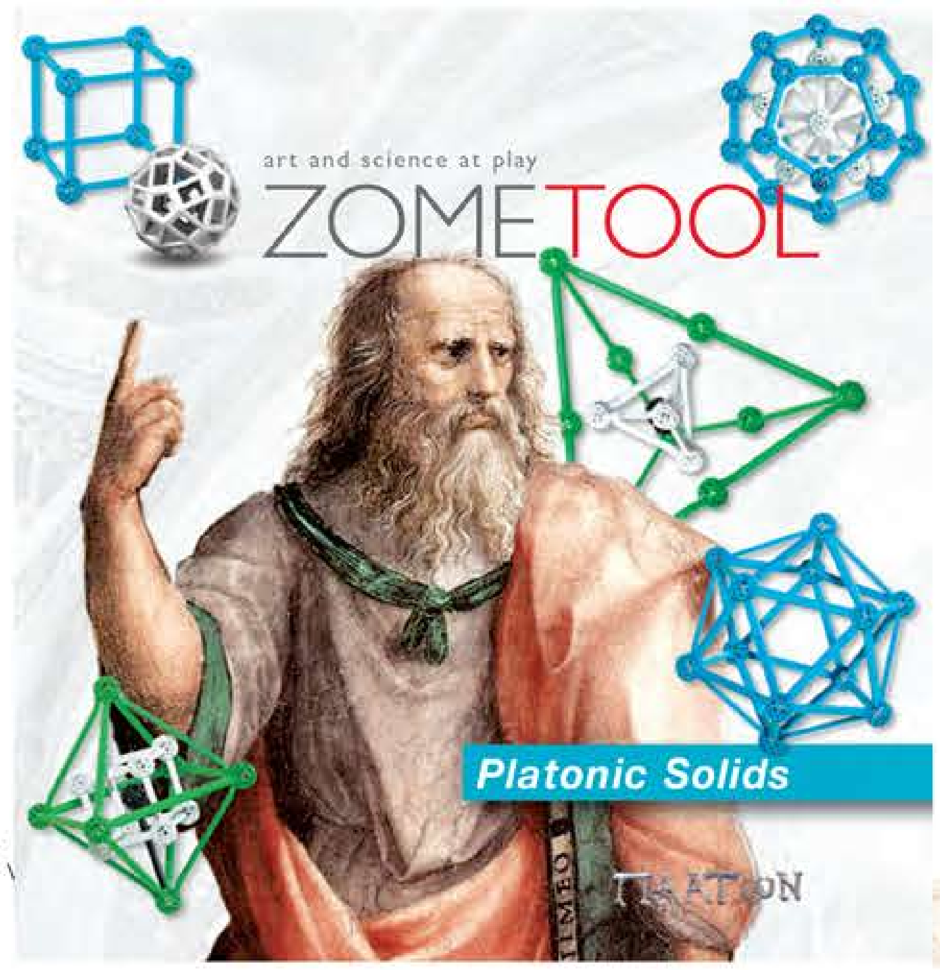
Thank you for using the Zometool! Please protect your investment and respect this magical tool: it's powerful, enlightening, fun, (expensive!) and you can break it.

Take it apart, don't break it apart. Squeeze balls off struts with your thumb and forefinger. You can practice using a Zometool "lollipop." The ball will eject from the strut with no danger of damaging either.

If it works, it works perfectly. The Zometool is mathematically precise. Although you may bend a strut to insert it in a tight spot, finished models are "stress-free:" no bending, twisting or fudging necessary!



- Number = space
- Infinite dimensions
- No time; only now



art and science at play

ZOMETOOL®

"Where there is matter, there is geometry."
(Johannes Kepler, Harmonices Mundi, 1619)

The five forms known as the Platonic Solids are found not only in mathematics, art and architecture, but also in the world around us: in nature they appear in crystals and many living organisms, such as protozoa and viruses.

These forms have been known to humans for thousands of years. The earliest representations — a series of intricately fashioned marble-like objects called the Ashmolean carvings — are over 5,000 years old.

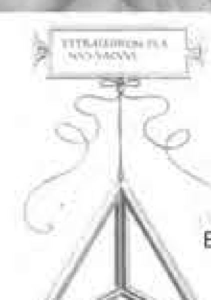
Although these carvings do not agree with the Platonic Solids in all details, they provide inspiring insight into our age-old fascination with the regular solids. The earliest pyramids were built around the same time and show ancient Egyptians' knowledge of the tetrahedron, cube and octahedron.

left and above: Radiolarians (protozoa), illustrations from Ernst Haeckel, 1879.

The Pythagoreans (ca. 550 BC) studied at least three of these solids: the tetrahedron, cube and dodecahedron and the Athenian Theaitetos (ca. 415-369 BC) may have been the first to prove that there can be only five convex regular polyhedrons.

The Platonic Solids are named after Plato (ca. 423-347), the Greek philosopher, mathematician and teacher whose writings greatly influenced Western thought. Plato was one of the first to describe the solids together and use them as the building blocks of an early "Theory of Everything." In his system, the properties of the four classical elements were represented by four regular solids — and the fifth (the dodecahedron) was "used for embroidering the constellations on the whole heaven". Later, Aristotle would associate it with the 'quintessential' element, ether.

tetrahedron	fire	plasma
cube (hexahedron)	earth	solid
octahedron	air	gaseous
icosahedron	water	liquid
dodecahedron	aether	a reference to the cosmos



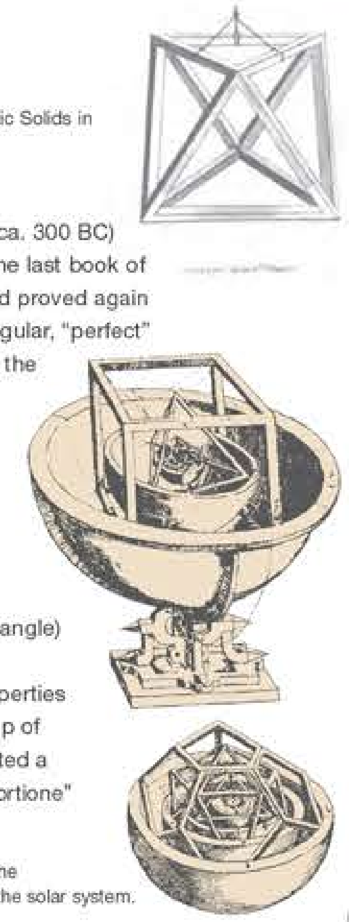
Leonardo da Vinci depicted all five Platonic Solids in "**Divina Proportione**".

Building on this knowledge, Euclid (ca. 300 BC) described the Platonic Solids in the last book of his seminal work "Elements" and proved again the existence of exactly five regular, "perfect" solids. He remarked that "No other figure, besides the said five figures, can be constructed by equilateral and equiangular figures equal to one another.

A simple summary shows that for each solid:

- all edges are of the same length
- all faces are made of the same regular polygon
- the same number of faces meet at each vertex
- all face edges meet at the same angle (dihedral angle)

With the onset of the Renaissance, the unique properties of these forms ignited the curiosity of a wider group of intellectuals: Leonardo da Vinci (1452 - 1519) created a series of illustrations for Luca Pacioli's "Divina Proportione"

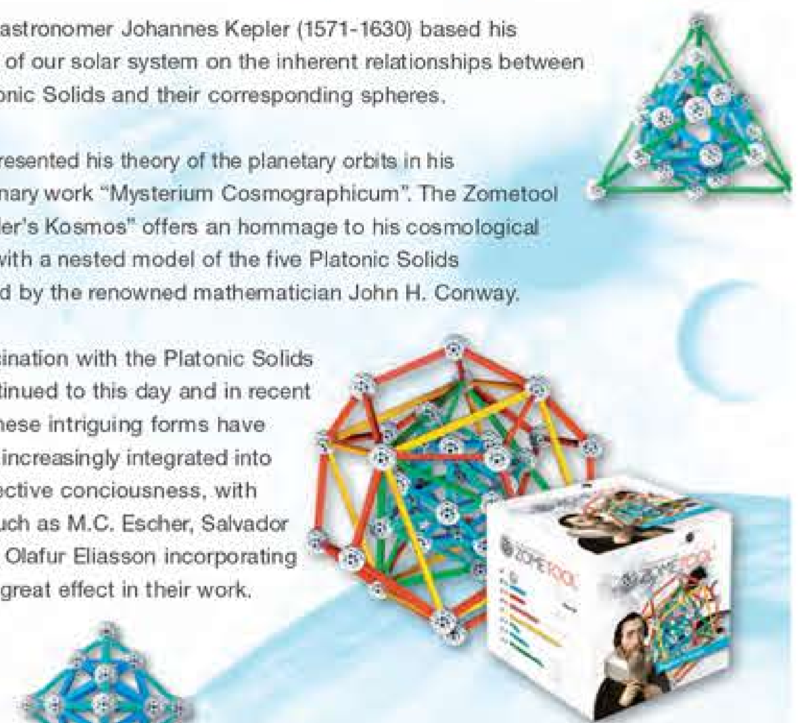


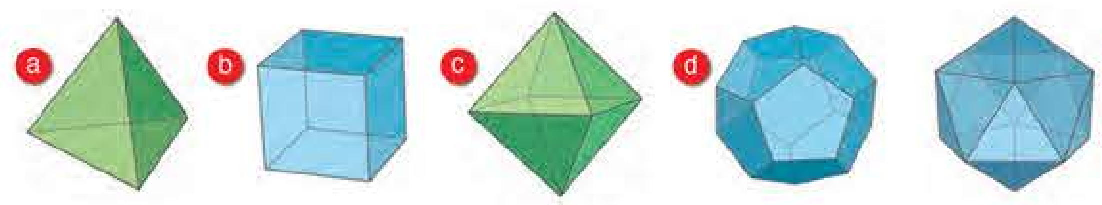
obove right: Kepler's illustrations of the Platonic Solids from the "**Mysterium Cosmographicum**, 1596 and right: his model of the solar system.

and the astronomer Johannes Kepler (1571-1630) based his concept of our solar system on the inherent relationships between the Platonic Solids and their corresponding spheres.

Kepler presented his theory of the planetary orbits in his revolutionary work "Mysterium Cosmographicum". The Zometool kit "Kepler's Kosmos" offers an hommage to his cosmological theory, with a nested model of the five Platonic Solids proposed by the renowned mathematician John H. Conway.

The fascination with the Platonic Solids has continued to this day and in recent times, these intriguing forms have become increasingly integrated into our collective conciousness, with artists such as M.C. Escher, Salvador Dali and Olafur Eliasson incorporating them to great effect in their work.





Defining characteristics of the five Platonic Solids

Platonic Solid	faces	2-D shape (polygon)	edges	vertices
a - tetrahedron	4	equilateral triangle	6	4
b - cube	6	square	12	8
c - octahedron	8	equilateral triangle	12	6
d - dodecahedron	12	regular pentagon	30	20
e - icosahedron	20	equilateral triangle	30	12

Each of the Platonic Solids shares a common centre point with 3 spheres:

- **Circumsphere**. All vertices of a solid lie on the inner surface of a sphere. These corners represent the most symmetrical distribution for that number of points on the surface of their circumsphere.
- **Insphere**. This is contained by the solid and touches the centre of each of its faces.
- **Midsphere**. This passes through the midpoint of each of the edges of the solid.

right: circumsphere and insphere of an octahedron. To construct any Platonic Solid within a sphere, the vertices must simply be positioned at an equal distance from each other.

right: the dual solid of a tetrahedron can be constructed by connecting the adjacent midpoints of each face.

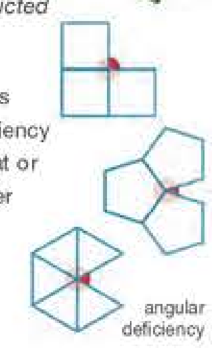
Duals

The Platonic Solids are also related to each other in several ways; one of the most important is the relationships of each solid with its dual. The vertices of each dual can be easily defined by placing a point in the center of each face of a polyhedron. By connecting each point to the next nearest point, the dual solid will materialise automatically. You will find a building guide for each solid and its dual in the 'Step by Step' section.

Why can there be only 5?

"No other figure, besides the said five figures, can be constructed by equilateral and equiangular figures equal to one another."

One way to prove the accuracy of Euclid's statement involves laying out polygons in a 2-D 'net' and using the angular deficiency between them to determine whether the net would remain flat or 'pop' into 3D. You can do this yourself using pencil and paper or Zometool components!



The Zometool is based on the 31-zone system, discovered by Steve Baer, Zomeworks Corporation, Albuquerque, NM USA